What is claimed is:

- 1. A honeycomb substrate, comprising:
 - an inner body having an inner skin and an array of inner webs defining an array of inner cells within the inner skin; and
 - an outer body having an outer skin formed concentric with the inner skin and an array of outer webs defining an array of radially-oriented triangle cells between the inner skin and the outer skin.
- 2. The honeycomb substrate of claim 1, wherein the array of inner cells comprises an array of square cells.
- 3. The honeycomb substrate of claim 2, wherein a length of the triangle cells along a radial direction is in a range from approximately 0.1 to 2 times a length of the square cells.
- 4. The honeycomb substrate of claim 2, wherein a thickness of the outer webs is greater than or equal to approximately 0.5 times a thickness of the inner webs.
- 5. The honeycomb substrate of claim 2, wherein the triangle cells are arranged in at least two layers between the inner skin and the outer skin.
- 6. The honeycomb substrate of claim 5, wherein an interface skin is formed between the two layers, the interface skin being concentric with the inner and outer skins.
- 7. The honeycomb substrate of claim 5, wherein a length of the triangle cells along a radial direction is in a range from approximately 0.1 to 2 times a length of the square cells.
- 8. The honeycomb substrate of claim 5, wherein a thickness of the outer webs is in a range from approximately 0.1 to 4 times a thickness of the inner webs.
 - 9. The honeycomb substrate of claim 1, wherein a thickness of the inner webs is less than or equal to approximately 0.15 mm.

- 10. The honeycomb substrate of claim 9, wherein fillets are formed at intersections of the outer webs and the outer skin.
- 11. The honeycomb substrate of claim 10, wherein a radius of the fillets is in a range from approximately 0.05 to 0.2 mm.
- 12. The honeycomb substrate of claim 1, wherein the array of triangle cells further comprises an array of radially-oriented diamond cells.
- 13. The honeycomb substrate of claim 12, wherein a length of the diamond cells along a radial direction is in a range from approximately 0.1 to 4 times a length of the square cells.
- 14. The honeycomb substrate of claim 1, wherein a hydraulic diameter of the triangle cells is equivalent to a hydraulic diameter of the inner cells.
- **15.** A honeycomb substrate, comprising:
 - an inner body having an inner skin and an array of inner webs defining an array of inner cells within the inner skin; and
 - an outer body having an outer skin formed concentric with the inner skin and an array of outer webs defining an array of radially-oriented quadrilateral cells between the inner skin and the outer skin, the quadrilateral cells having a length along a circumferential direction no greater than a length of the inner cells.
- 16. The honeycomb substrate of claim 15, wherein the array of inner cells comprises an array of square cells.
- 17. The honeycomb substrate of claim 15, wherein the length of the quadrilateral cells along the circumferential direction is no less than 0.5 times the length of the inner cells.
- 18. The honeycomb substrate of claim 15, wherein a length of the quadrilateral cells along a radial direction is in a range from approximately 0.1 to 2 times the length of the inner cells.

- 19. The honeycomb substrate of claim 15, wherein a thickness of the inner webs is less than or equal to approximately 0.15 mm.
- **20.** The honeycomb substrate of claim **15**, wherein a thickness of the inner skin is greater than a thickness of the inner webs.
- 21. The honeycomb substrate of claim 15, wherein a thickness of the outer skin is greater than twice a thickness of the inner webs.
- 22. The honeycomb substrate of claim 19, wherein fillets are formed at intersections of the outer webs and the outer skin.
- 23. The honeycomb substrate of claim 22, wherein a radius of the fillets is in a range from approximately 0.05 to 0.2 mm.
- **24.** A honeycomb substrate, comprising:
 - an inner body having an inner skin and an array of inner webs defining an array of inner cells within the inner skin; and
 - an outer body having an outer skin formed concentric with the inner skin and an array of first outer webs arranged parallel to a diametrical axis of the inner body and an array of second outer webs arranged in a crisscross pattern, the first and second outer webs intersecting to define an array of triangle cells between the inner skin and the outer skin.
- **25.** The honeycomb substrate of claim **24**, wherein the array of inner cells comprises an array of square cells.
- 26. The honeycomb substrate of claim 25, wherein a length of the triangle cells along the diametrical axis ranges from 0.1 to less than 2 times a length of the square cells.
- 27. The honeycomb substrate of claim 25, wherein a thickness of the first and second webs is greater than 0.5 times a thickness of the inner cells.
- 28. The honeycomb substrate of claim 25, wherein a thickness of the inner webs is less than or equal to approximately 0.15 mm.

- **29.** An extrusion die assembly for making a double-skin honeycomb structure, comprising:
 - an inner cell forming die having a central region and a peripheral region, the central region comprising an array of inner slots cut to define an array of inner pins and an array of central feedholes in communication with the array of inner slots, the peripheral region comprising an array of peripheral feedholes;

a skin forming mask mounted coaxially with the inner cell forming die; and at least one outer cell forming die mounted coaxially with and interposed between the inner cell forming die and the skin forming mask, the outer cell forming die having an array of outer slots cut to define an array of outer pins, the outer pins being spaced radially from the central region to define an inner skin slot, the inner skin slot being in selective communication with the peripheral feedholes, the outer pins being spaced radially from the skin forming mask to define an outer skin slot, the outer skin slot being in selective communication with the peripheral feedholes through an opening in the outer cell forming die.

- 30. The extrusion die assembly of claim 29, further comprising an outer skin reservoir defined between the outer cell forming die and the skin forming mask.
- 31. The extrusion die assembly of claim 30, wherein the outer skin reservoir is in communication with the outer skin slot and the opening in the outer cell forming die.
- 32. The extrusion die assembly of claim 31, wherein a volume of the outer skin reservoir is adjustable to control rate of flow of extrudable material to the outer skin slot.
- 33. The extrusion die assembly of claim 31, further comprising an inner skin reservoir defined between the inner cell forming die and the outer cell forming die.
- 34. The extrusion die assembly of claim 33, wherein the inner skin reservoir is in communication with the inner skin slot and the peripheral feedholes.
- 35. The extrusion die assembly of claim 34, wherein a volume of the inner skin reservoir is adjustable to control rate of flow of extrudable material to the inner skin slot.

- 36. The extrusion die assembly of claim 35, wherein the inner skin reservoir is in communication with the outer skin reservoir through the opening in the outer cell forming die.
- 37. The extrusion die assembly of claim 34, wherein a volume of the inner skin reservoir relative to a volume of the outer skin reservoir is such that flow velocity in the outer slots gradually increases from the inner skin slot to the outer skin slot.
- 38. The extrusion die assembly of claim 29, wherein the opening comprises a plurality of orifices outboard of the outer slots.
- 39. The extrusion die assembly of claim 29, wherein the opening comprises a radial extension of the outer slots.
- 40. The extrusion die assembly of claim 29, wherein the outer pins are oriented radially with respect to a center of the inner cell forming die.
- 41. The extrusion die assembly of claim 40, wherein the inner pins have a square cross-section.
- 42. The extrusion die assembly of claim 40, wherein the array of outer pins comprises pins having a triangle cross-section.
- 43. The extrusion die assembly of claim 40, wherein the array of outer pins comprises pins having a diamond cross-section.
- **44.** The extrusion die assembly of claim **40**, wherein the outer pins have a quadrilateral cross-section.
- 45. The extrusion die assembly of claim 29, wherein a plurality of outer cell forming dies are interposed between the inner cell forming die and the skin forming mask.
- **46.** The extrusion die assembly of claim **45**, wherein an intermediate skin slot is defined between adjacent outer cell forming dies.